Equations

Any mathematical statement which states that Left Hand side is equals to Right hand side

$$LHS = RHS$$

eg $2x+3 = 5x^2 - 7x$

eg $3x^2 - 8x + 9 = 0$

eg $x^2 + y^2 = 16$

Degree of Equation

Highest power of variable is known as Degree of Equation

Eg $4x^2 + 5x + 3 = 0$

Degree = 2

Eg $2x^3 + 8x + 9 = 0$

Degree = 3

Eg $5x + 7y + 6 = 0$

Degree = 1

Types of Equation on the Basis of Degree

Linear	Quadratic	Cubic
$\Downarrow$	$\Downarrow$	$\Downarrow$
Degree 1	Degree 2	Degree 3

Solutions of The Equation

The value of variable which satisfy the Equation is known as Root of Equation or solution of The equation

Eg $\boxed{3x + 7 = 5x + 3}$

$x = 2$ is a solution of This Equation because if we put $x = 2$, LHS & RHS will become Equal

$LHS = 3(2) + 7$
$= 13$

$RHS = 5(2) + 3$
13

$LHS = RHS$

Lineaar Equation

In one variable → $\boxed{ax+b=0}$

In Two variable → $\boxed{ax+by+c=0}$

eg $3x-8=5$

$3x=5+8$

$3x=13$

$x=\frac{13}{3}$

eg $\frac{x}{2}+\frac{x}{3}=5$

$\frac{3x+2x}{6}=5$

$\frac{5x}{6}=5$

$5x=30$

$x=\frac{30}{5}$

$x=6$

Methods of Solving Linear Equation in two variables

Substitution method

Elimination method

=> Substitution method

-> Two Equations will be given

-> find the value of x in terms of y in both Equation

-> Equate Two Equation

eg $x + 3y = 7$

$2x + y = 4$

find x & y

Sol:

$x + 3y = 7$ & $2x + y = 4$

$\Downarrow$ $\Downarrow$

$x = 7 - 3y$ $2x = 4 - y$

$x = \dfrac{4 - y}{2}$

Equate

$$7 - 3y = \frac{4 - y}{2}$$

$$2(7 - 3y) = 4 - y$$

$$14 - 6y = 4 - y$$

$$14 - 4 = -y + 6y$$

$$10 = 5y$$

$$y = 2$$

now $x = 7 - 3y$

$$x = 7 - 3(2)$$

$$x = 1$$

⇒ $\boxed{\text{Elimination Method}}$

→ Two equations are given
→ select one variable & make its coefficient same in both equation
→ Then eliminate that variable using addition or subtraction

Eg $2x + 3y = 5$
$5x + 2y = 7$
find x & y

Sol: in given two equation i will eliminate variable 'y'

$$[2x + 3y = 5] \times 2$$
$$[5x + 2y = 7] \times 3$$

$$\begin{bmatrix} 4x + 6y = 10 \\ 15x + 6y = 21 \end{bmatrix}$$

Subtract second from first

$$\begin{array}{r} 4x + 6y = 10 \\ -15x - 6y = -21 \\ \hline -11x = -11 \end{array}$$

$$x = 1$$

Now $4x + 6y = 10$

$$4(1) + 6y = 10$$

$$6y = 10 - 4$$

$$6y = 6$$

$$y = 1$$

Quadratic Equation

$ax^2 + bx + c = 0$ where $a \neq 0$

It can have maximum Two Roots which are generally Denoted by 'α' & 'β'

Sum of Two Roots $= \alpha + \beta = -\dfrac{b}{a}$

Product of Two Roots $= \alpha \cdot \beta = \dfrac{c}{a}$

$$\alpha^2 + \beta^2 = (\alpha+\beta)^2 - 2\alpha\beta$$

$$\alpha^3 + \beta^3 = (\alpha+\beta)(\alpha^2+\beta^2-\alpha\beta)$$

$$\left[(\alpha+\beta)^2-(\alpha-\beta)^2 = 4\alpha\beta\right]$$

when one Root is the reciprocal of other Root Then $a=c$

when sign of two roots is opposite but magnitude is same Then

$b=0$

If one irrational Root is $m+\sqrt{n}$ Then other irrational Root will be $m-\sqrt{n}$

If Sum of Roots &
Product of Roots is given

Then Quadratic Equation

$x^2 - (\text{Sum of Roots})x + \text{Product} = 0$

Q find Q.E. whose Roots are 3 & 8

Sol: Sum of Roots $= 3 + 8 = 11$

Product of Roots $= 3 \times 8 = 24$

Q.E. will be

$x^2 - 11x + 24 = 0$

Methods of Solving Quadratic Equation

Factorization method

Quadratic formula

$\Rightarrow$ Factorization method
(middle term splitting)

$\rightarrow$ Calculate $a \times c$

$\rightarrow$ find two factor of ac such that their sum of Difference is Equal to b

eg $2x^2+11x+14=0$

find x

Sol: $a=2,\ b=11,\ c=14$

$ac = 2\times 14 = 28$

1×28 | 2×14 | 4×7

$1+28=29$

$1-28=-27$

$2+14=16$

$2-14=-12$

$4+7=11$

so $2x^2+11x+14=0$

$2x^2+7x+4x+14=0$

$x(2x+7)+2(2x+7)=0$

$(x+2)(2x+7)=0$

Now

$x+2=0$

$\boxed{x=-2}$

&

$2x+7=0$

$\boxed{x=-\frac{7}{2}}$

=> Quadratic formula

-> find a, b & c

-> Apply formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Eg $2x^2 + 11x + 14 = 0$

$a = 2, b = 11, c = 14$

$$x = \frac{-11 \pm \sqrt{(11)^2 - 4(2)(14)}}{2(2)}$$

$$x = \frac{-11 \pm \sqrt{9}}{4}$$

$$x = \frac{-11 \pm 3}{4}$$

now

$$x = \frac{-11 + 3}{4}$$

$$x = \frac{-8}{4}$$

$$\boxed{x = -2}$$

&

$$x = \frac{-11 - 3}{4}$$

$$x = \frac{-14}{4} \Rightarrow \boxed{x = \frac{-7}{2}}$$

Discriminant = D

$\Downarrow$

$D = b^2 - 4ac$

$\Downarrow$

It will tell nature of Roots

If $D = b^2 - 4ac > 0$
Roots are Real & Different

If $D = b^2 - 4ac = 0$
Roots are Real & Equal

If $D = b^2 - 4ac < 0$
Roots are Not Real

If D is positive Perfect Square
Then Root can be Rational

If D is Not Perfect square
Then Roots can Irrational

Cubic Equation

$ax^3 + bx^2 + cx + d = 0$ where $a \neq 0$

It can have maximum 3 Roots α, β & γ

$$\alpha + \beta + \gamma = \frac{-b}{a}$$

$$\alpha\beta + \beta\gamma + \alpha\gamma = \frac{c}{a}$$

$$\alpha\beta\gamma = \frac{-d}{a}$$

Cubic Equation

$$x^3 - (\alpha + \beta + \gamma)x^2 + (\alpha\beta + \beta\gamma + \alpha\gamma)x - \alpha\beta\gamma = 0$$